Nonlocal transport in Toroidal Fusion Devices

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- Particle transport is nonlocal in many fusion devices even in the presence of small field perturbations

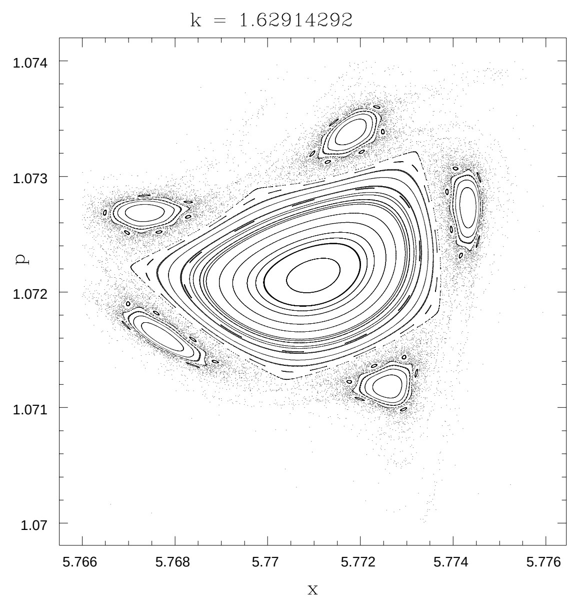
- We examine the early stages of disruptions in ITER and ASDEX, discharges with a spectrum of TAE modes in NSTX and DIII-D, and the normal state of saturated tearing modes in RFX

- Orbit resonances can produce long time correlations and traps for particle trajectories at perturbation amplitudes much too small for the orbits to be represented as uniformly chaotic.

- The existence and nature of subdiffusive transport is found to depend on the nature of the mode spectrum and frequency as well as the mode amplitudes.

Subdiffusion

$$(\delta r)^2 = Dt^p \qquad p < 1$$



Perturbations produce complicated structures in orbit trajectories, leading to particle trapping and causing long term Lévy flights.
The traps can cause a particle to spend very long time at a fixed radius, leading to subdiffusion and nonlocality. Distributions dr and dt
The trap shown is from the Standard map,
but the nature of the traps is universal. R. White, Chaos [8] 757 (1998)

• It is well known that transport is not always diffusive in situations involving stochastic magnetic fields (Isichenko, Spineanu, Vlad, Balescu). These publications evaluate the transport in terms of characterizations of the field, such as correlation lengths and times.

Model stochastic field

$$\vec{B} = \sum_{k,\sigma} \delta B(k) e^{\sigma}(k) e^{i(\vec{k}\cdot\vec{r} + \phi_k^{\sigma})}$$

$$\vec{k} = \frac{2\pi}{N}\left(\frac{n_x}{L_\perp}, \frac{n_y}{L_\perp}, \frac{n_z}{L_\parallel}\right),$$

Self similar isotropic spectrum

$$\delta B(k) = \frac{A}{(k_\perp^2 L_\perp^2 + k_\parallel^2 L_\parallel^2)^{\gamma/4+1/2}}$$

σ polarization, ϕ random phases.

The results give transport related to the Kubo number, and the Lyapunov exponent, and particle orbit properties such as the gyro radius and collision rates.

$$K = \frac{\delta B}{B} \frac{L_{\parallel}}{L_{\perp}}, \quad < dx^2 > \sim e^{Lt}, \quad \rho, \quad \nu$$

For $0.3 < K < 1$, D has quasilinear scaling

$$D = CK^2 L_{\perp}^2 / L_{\parallel}$$

• But it is not clear what this means for typical plasma discharges. What are these numbers in a typical discharge?
It takes as much work to evaluate these parameters as to investigate the transport directly.

• Furthermore often the perturbation spectrum is very sparse, not leading to well defined mean values for these parameters.
The model field is not a good representation of the discharge.

Particles are followed using the guiding center code Orbit. The guiding center Hamiltonian is

$$H = \rho_{\|}^2 B^2/2 + \mu B + \Phi,$$

where $\rho_{\|} = v_{\|}/B$ is the normalized parallel velocity, μ is the magnetic moment, and Φ the electric potential. The field magnitude B and the potential may be functions of the poloidal flux ψ_p, the poloidal angle θ and also the toroidal angle ζ if axisymmetry is broken.

The equations of motion in Hamiltonian form are

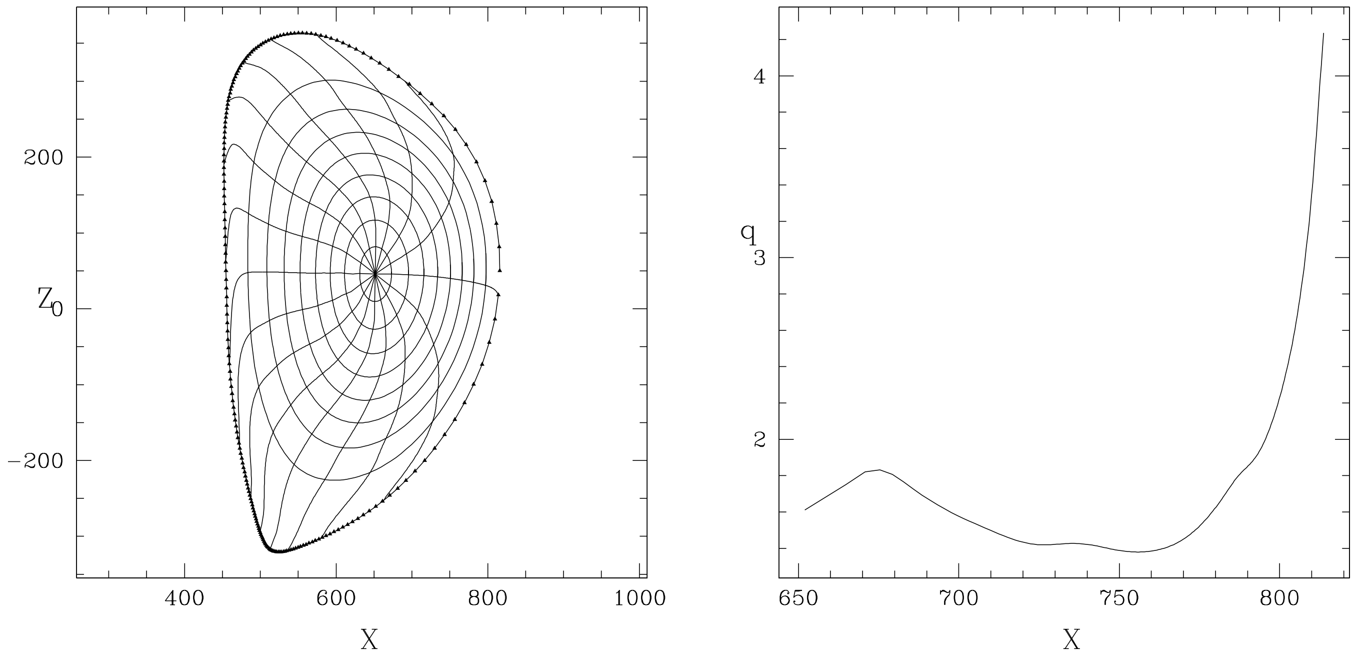
$$\dot{\theta} = \frac{\partial H}{\partial P_\theta} \qquad \dot{P}_\theta = -\frac{\partial H}{\partial \theta}$$
$$\dot{\zeta} = \frac{\partial H}{\partial P_\zeta} \qquad \dot{P}_\zeta = -\frac{\partial H}{\partial \zeta},$$

where canonical momenta are

$$P_\zeta = g\rho_{\|} - \psi_p, \qquad P_\theta = \psi + \rho_{\|} I,$$

and ψ is the toroidal and ψ_p the poloidal flux, with $d\psi/d\psi_p = q(\psi_p)$, the field line helicity. Functions $g(\psi_p)$ and $I(\psi_p)$ give the toroidal and poloidal field magnitudes.

ITER



We consider an advanced scenario equilibrium for ITER.
Case 40000B11 at 250 sec, with strongly reversed shear.
Equilibrium (left) and q profile (right), B = 50.1 kG on axis.

Disruption simulations (Hopcraft,Waddell) typically show a large 2/1 island and a smaller 3/2 island driving a 5/3 island to overlap with the 2/1 producing a stochastic domain.

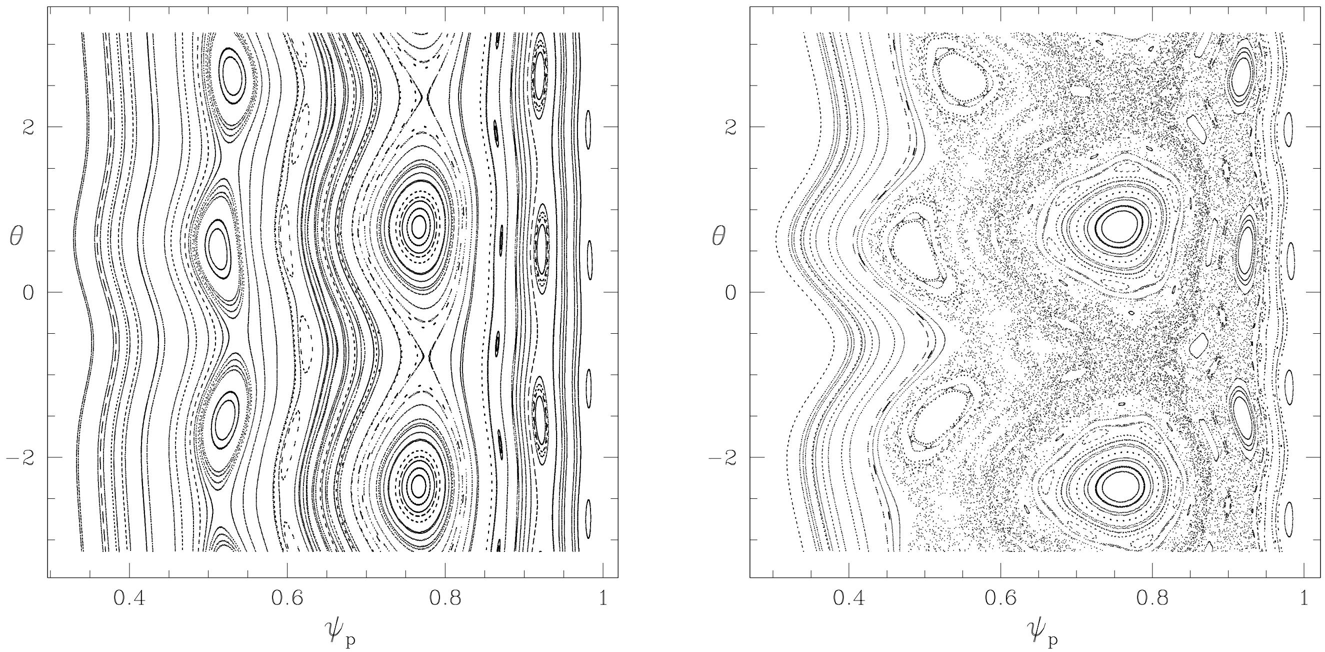
Perturbations associated with tearing modes are added
to the equilibrium, producing resonance islands at rational surfaces where the field helicity q equals the rational number m/n.
The form of the perturbation is

$$\delta\vec{B} = \nabla \times \alpha(\psi_p, \theta, \zeta, t)\vec{B}$$

$$\alpha(\psi_p, \theta, \zeta, t) = \sum_{m,n} \alpha_{mn}(\psi_p) sin(n\zeta - m\theta - \omega t)$$

The perturbations are used at a given amplitude to study their effect on the particle transport.

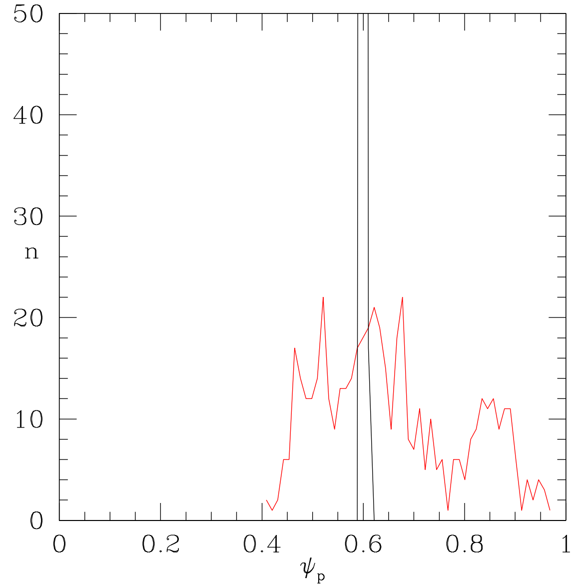
Thermal Ions in a pre disruption ITER configuration



A kinetic Poincaré plot of 1 keV deuterium trajectories with $\mu = 0$. Tearing modes with $m/n = 3/2$, $2/1$, $3/1$, and $4/1$ with maximum amplitudes $\alpha_{2,1} = 2 \times 10^{-3} A$, $\alpha_{2,3} = 2 \times 10^{-4} A$, $\alpha_{3,1} = 4 \times 10^{-4} A$, $\alpha_{4,1} = 4 \times 10^{-4} A$, showing $A = 0.3$ and $A = 1$.

The 5/3 and 5/2 resonances are visible and there are also higher order Fibonacci sequence islands present.

Initial and final ion densities



An example of initial and final ion densities

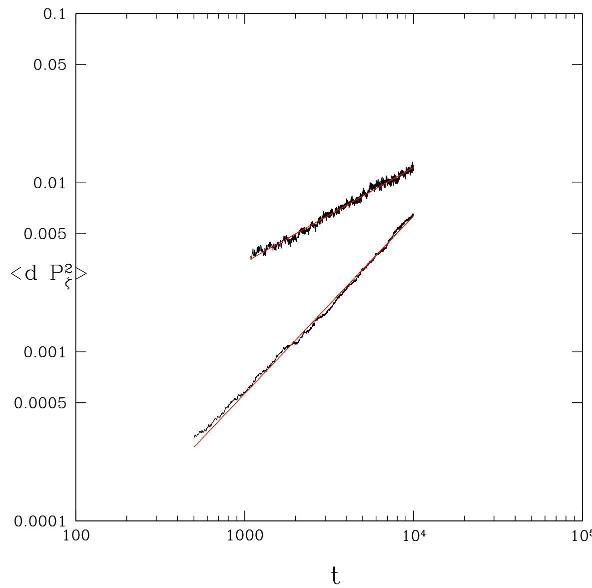
$A = 0.9$ for collision rate $\nu T = 0.01$

Here T is the on-axis transit time.

As long as ν is small enough that the mean free path is many toroidal transits ($\nu T << 1$) the transport type is independent of ν.

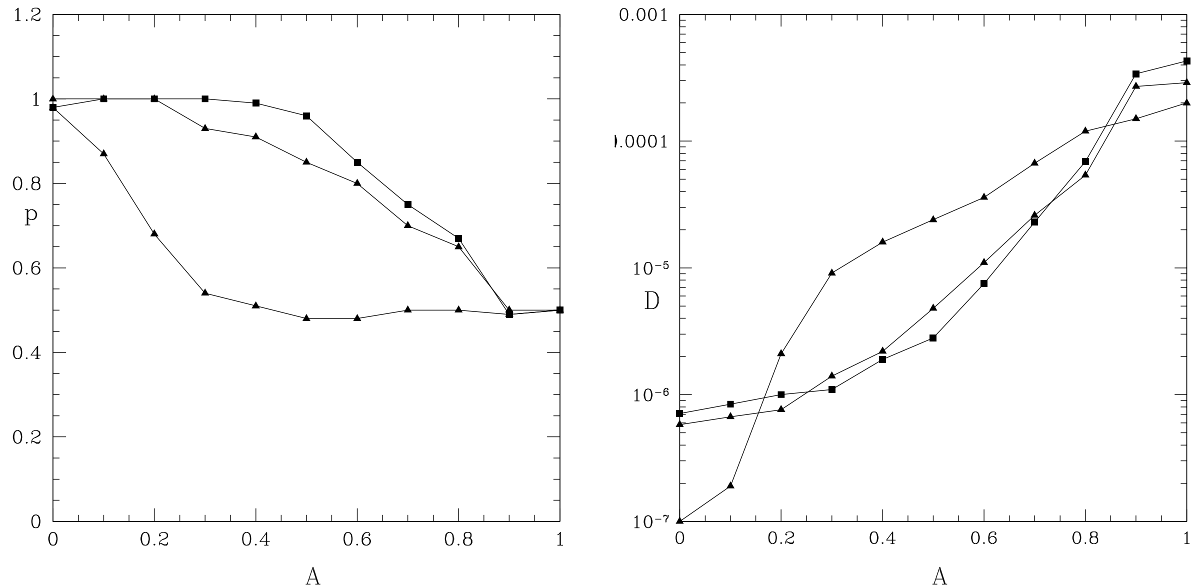
For accurate values the transport must not reach the boundaries.

ITER pre disruption



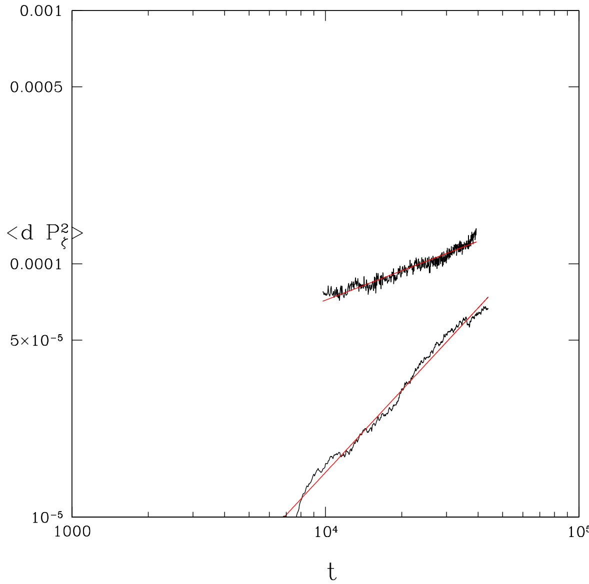
Asymptotic ITER ion transport fit by $<\delta P_\zeta^2>= Dt^p$,
showing the evaluation of p as the slope of line fit to the data.
The plots are for $A = 0.1$, $\nu T = 0.1$, giving $p = 1.0$ (lower)
and $A = 0.8$, $\nu T = .01$, giving $p = 0.5$ (upper)
T is on-axis transit time, about 0.1 msec.
Classical subdiffusion is given by $p = 0.5$

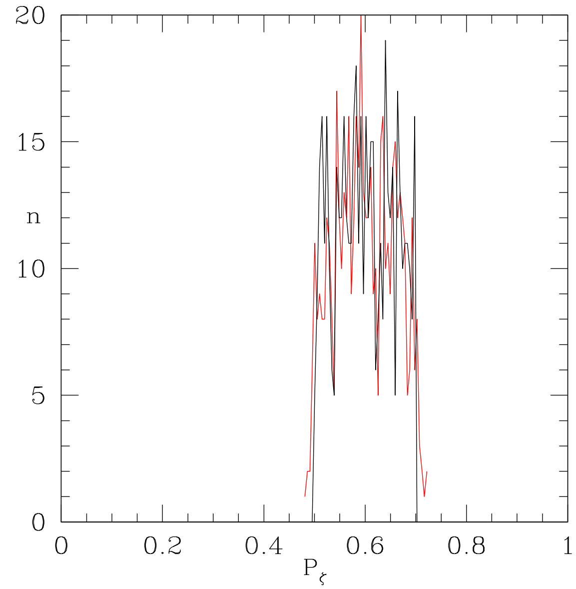
Asymptotic ITER ion transport



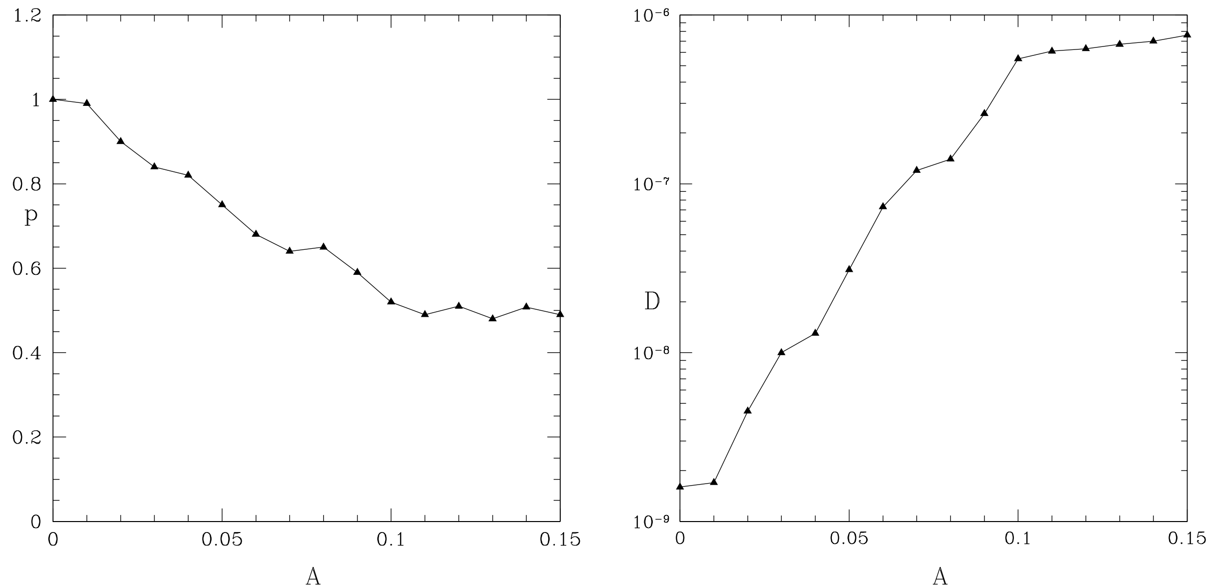
Asymptotic ITER ion transport fit by $<\delta P_\zeta^2> = Dt^p$.
Plots of p (left) and D (right) vs A.
The lowest curves for p are for 1 keV ions with $\nu T = 10^{-2}$ and 10^{-1} (triangles), and the upper curve is for 10 keV ions with $\nu T = 10^{-2}$ (squares), with T the toroidal transit time.

ITER electron transport



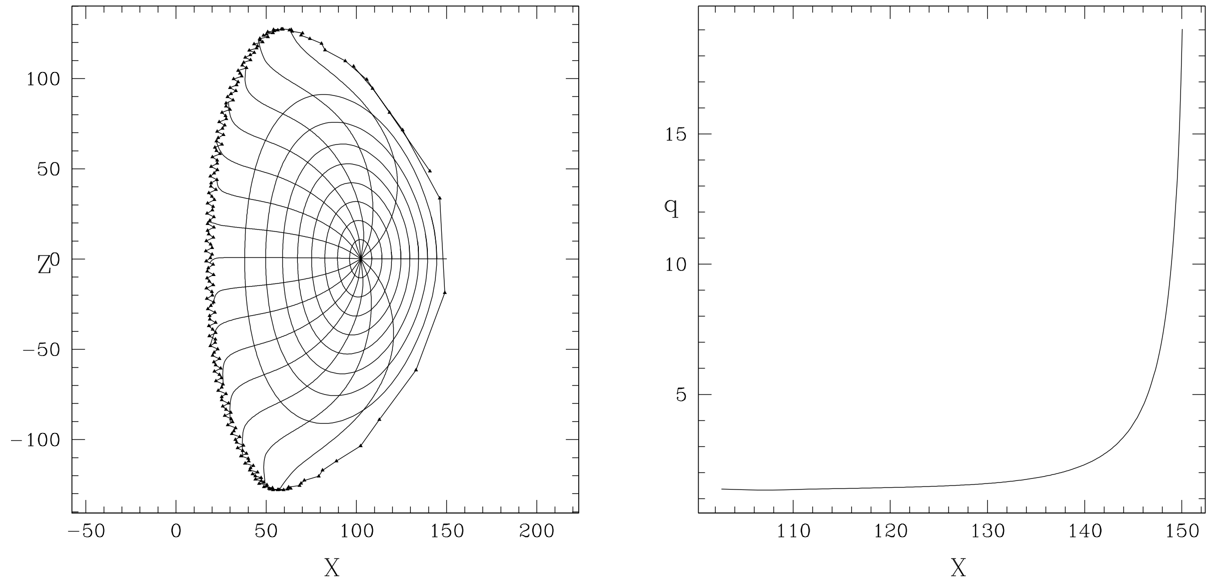


Determination of 10 keV electron transport in ITER
mode amplitudes of A=0 (p = 1, lower) and A = 0.15 (p = .5, upper)
Initial and final distributions after 10^4 toroidal transits for $A = 0.1$
At $A = 0$ $D = 10^{-9}$. Using the transit time of $7 \times 10^{-7} sec$, and for electrons P_ζ is approximately the poloidal flux, and using $\rho = 7 \times 10^{-3} cm$, this value agrees with the Pfirsch-Schl$\ddot{u}$ter value of diffusion $D = \nu q^2 \rho^2 / 2$, giving approximately 7 cm^2/sec.



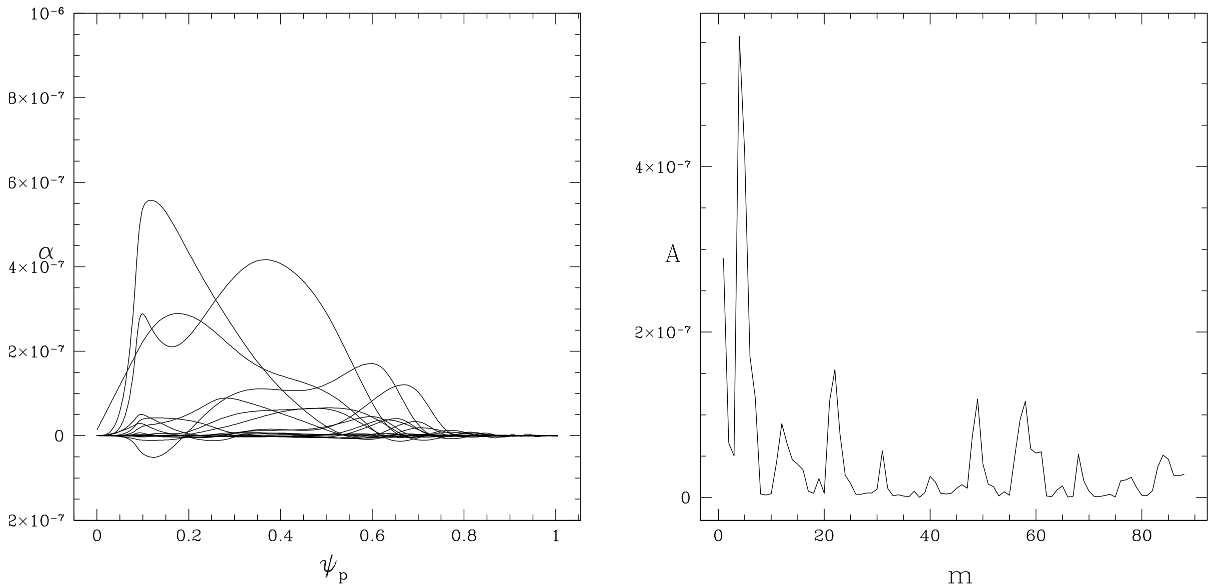
Asymptotic ITER electron transport fit by $<\delta P_\zeta^2>= Dt^p$.
Plots of p and D vs A for 10 keV electrons with $\nu T = 10^{-1}$.
Subdiffusion obtained for very small amplitudes A.
The constant D increases monotonically with amplitude A,
but much more slowly after subdiffusion is attained.

NSTX



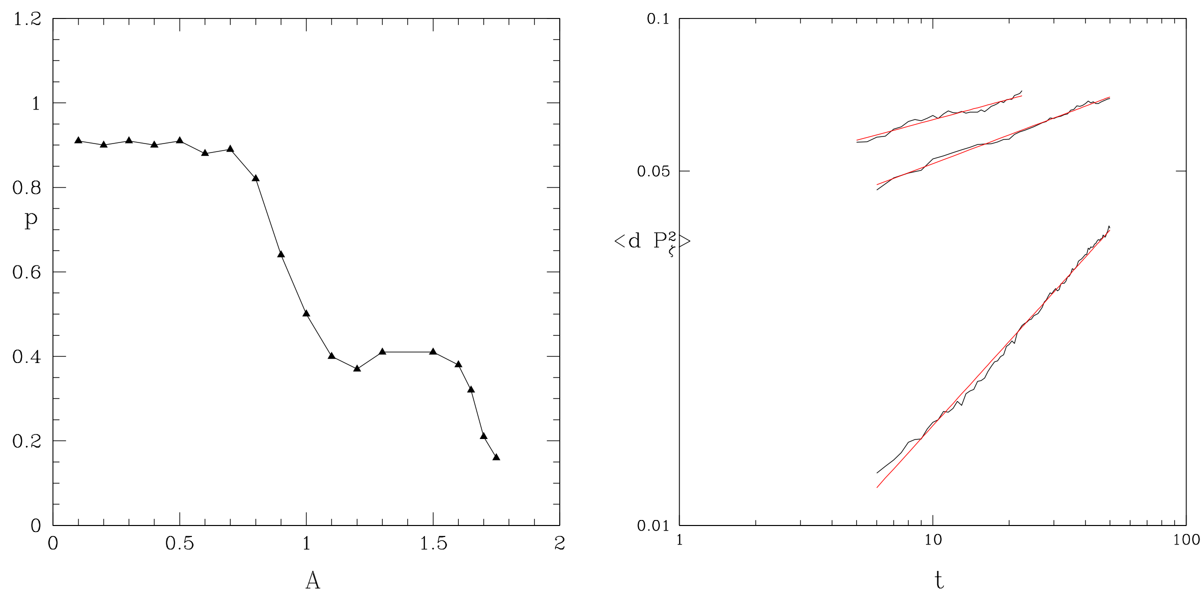
Discharge 141711 in NSTX at a time of 470 msec.
A spectrum of TAE modes observed and analyzed using NOVA, giving eigenfunctions $\alpha_{mn}(\psi_p)$

NSTX



Discharge 141711 in NSTX at a time of 470 msec.
The frequency of the modes is around 100 kHz, and the transit time for a 2 keV electron is 2×10^{-7} sec, so an electron explores 50 toroidal transits in one mode period, enough to explore the field structure.
Ion velocity is too small to explore the field structure in one mode period, and ion transport was found to be diffusive for all amplitude values. Mode harmonics for TAE modes in NSTX (left) and the mode amplitudes as a function of the poloidal mode number, m (right).

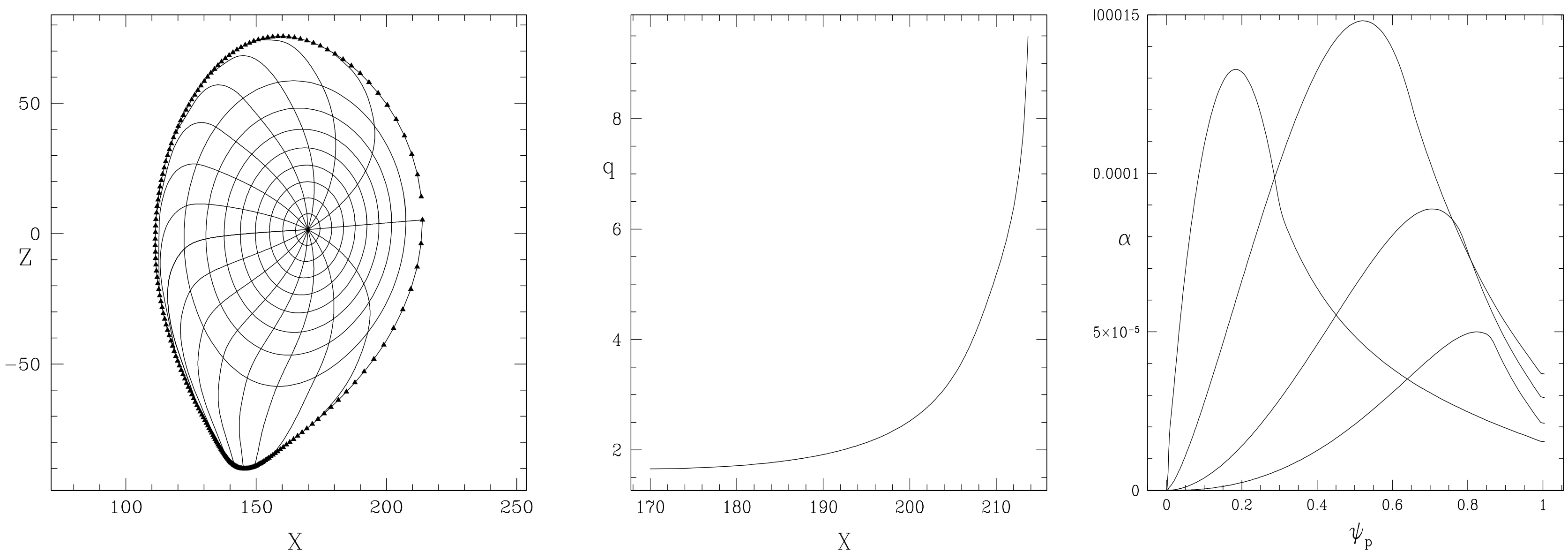
NSTX



Asymptotic transport fit by $< \delta P_\zeta^2 >= Dt^p$ for 2 keV electron transport for TAE modes in NSTX. Plot of p vs A (left) , and plots for A $=$ 1.5 with $p = .41$ (lower), A $=$ 1.7 with $p = .21$ (middle), and A $=$ 1.75 with $p = .16$ (upper).

Electrons are seen to make a fairly rapid transition to subdiffusive behavior at about the experimentally observed mode amplitudes, $A = 1$.

ASDEX

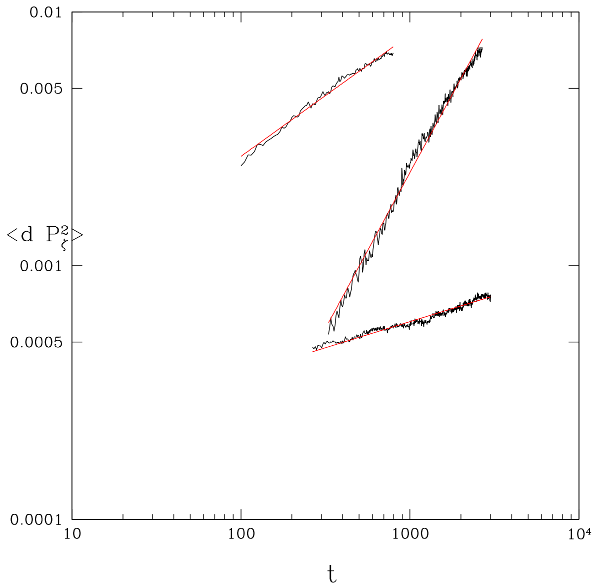


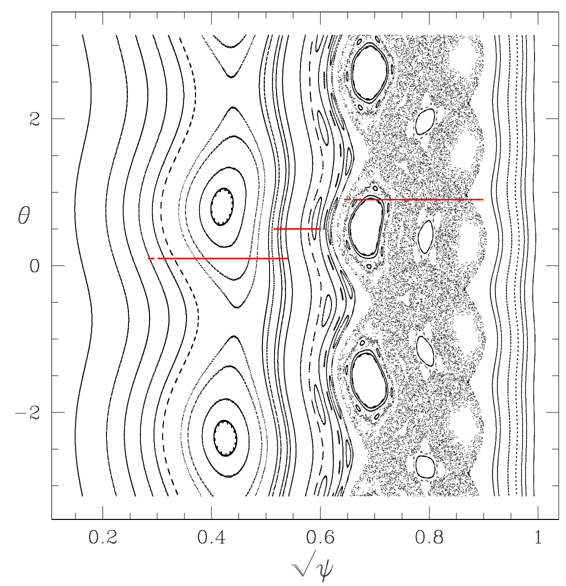
ASDEX Upgrade (AUG) the pre-disruptive phase of the L-mode, high density shot 30984, at t=1.398 seconds

Experimentally measured harmonics, with $m = 2, 3, 4, 5$, all with $n = 1$.

Mode frequency was 1.7 kHz

ASDEX electrons





AUG electron diffusion. For $\sqrt{\psi} = 0.4$ the transport was superdiffusive with $p = 1.23$, shown in the middle curve on the left.
Final particle distribution shown in the Poincaré plot at $\theta = 0.1$.
At $\sqrt{\psi} = 0.6$ subdiffusive transport with $p = 0.2$, with the final particle distribution at $\theta = 0.2$.
$\sqrt{\psi} = 0.8$ is within the stochastic domain existing outside the $\sqrt{\psi} = 0.7$ surface and the transport was subdiffusive with $p = 0.5$, with the final particle distribution at $\theta = 0.3$.

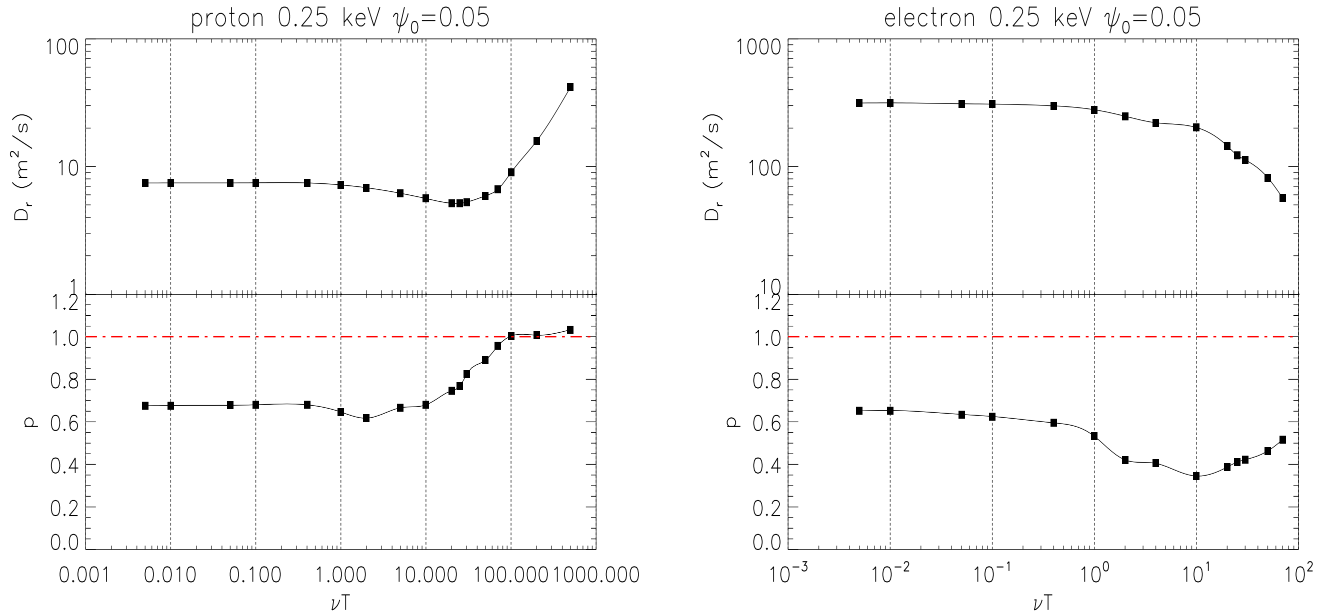
RFX

In the field of RFPs, examination of ion transport in the RFX, using a guiding center code, has shown that the pinch effect is actually a manifestation of the nonlocal subdiffusive motion of particles in the chaotic field produced by saturated tearing modes (Spizzo *et al.*, 2007). Particles do not move in a diffusive way, but follow the chaotic field lines across the original equilibrium flux surfaces; the resulting flight statistics are of the Lévy type.

The RFP is unique among confinement devices in that it possesses a chaotic field which is well known and relatively stable, thus providing an excellent test of theoretical models. But in addition, the tearing mode spectrum produces a magnetic field structure that is only slightly above stochastic threshold, so a random phase approximation is not valid; there still exist long scale correlations.

Rechester-Rosenbluth transport is not valid.

RFX



Proton and electron transport parameters in the RFX

Uniformly subdiffusive for realistic collision frequencies

The collision frequency for an ion of energy 250 eV corresponds to a collision time of 2.5 transits, $\nu T = 0.4$.

Flight time Model

Only passing particles experience the complex topology of the field, trapped particles simply execute banana diffusion.
Time spent passing we denote as a flight.
The flight time distribution is given by the pitch angle scattering operator, given by

$$\lambda' = \lambda(1 - \nu dt) \pm \sqrt{(1 - \lambda^2)\nu dt},$$

where ν is the collision frequency, $\lambda = v_{\|}/v$, with $v_{\|}$ the velocity parallel to $\vec{B}$ and dt the numerical time step. A numerical simulation shows that this operator determines a flight time distribution, defined as the time between changes of the sign of the pitch, of

$$\psi(t) = \frac{a}{(t_0 + t)^{1.4}},$$

normalized to 1 in $(0, \infty)$.
Changing the collision frequency simply changes the time scale, and thus the values of t_0 and a, but not the large t behavior.

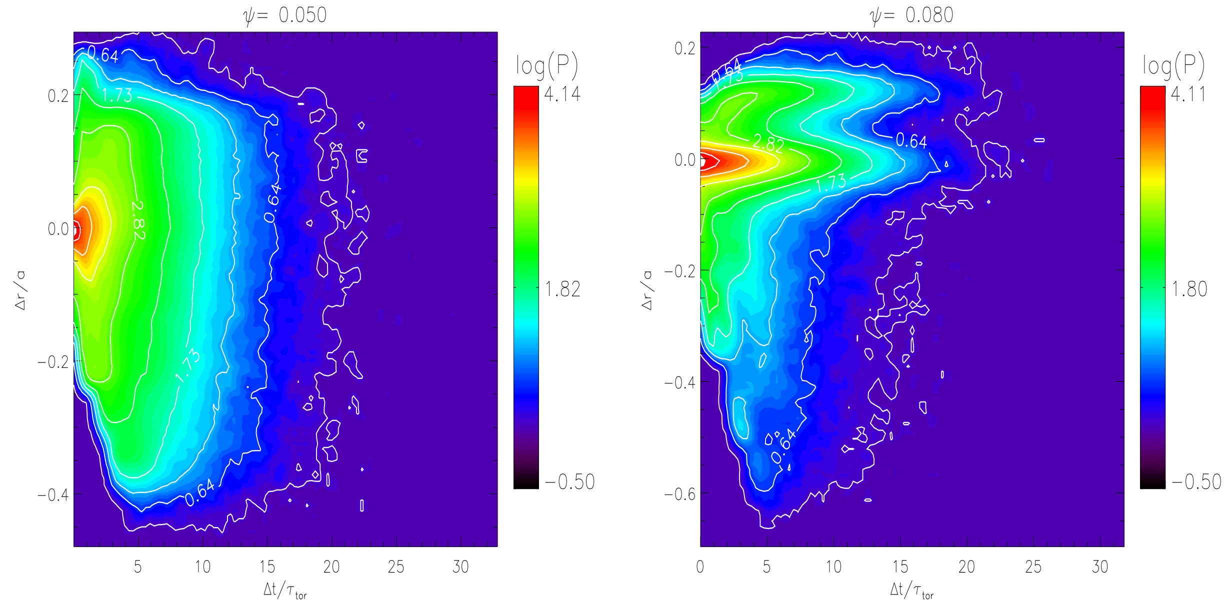
Flight time distribution truncated at small t by the trapped particle bounce frequency, assuming $t_0 \ll 1/\nu \ll 1$.
In the RFP, a typical value of the bounce frequency is approximately the on-axis toroidal transit time
The collision frequency for an ion of energy 250 eV corresponds to a collision time of 2.5 toroidal transits.
A trapped particle changes sign of pitch during each bounce, independent of the collisions.

Exact behavior near small t is not important, since there are no flights of very short duration, it is the asymptotic large t behavior that matters.
Note that $< t >$ is infinite.

Only passing particles experience the complex topology of the field, trapped particles simply diffuse following banana diffusion.

To make a tractible model, perform poloidal averages,
so transport is in one dimension, the minor radius only.
Introduce Lévy flight probability $p(r', r - r', t - t')$
Initial position and time r', t', with r, t final radius and time.
Determine p using the guiding center code.
Launch at r' and follow until the pitch changes sign
Record the new position r and the flight time $t - t'$.
Model takes account of the local variation of the level of stochasticity,
and finite boundaries of the plasma.

Simple models using parameters describing the field configuration
(fractional derivatives, etc) cannot account for the spatial variation.
Probability vanishes for large distance and small time,
reflecting the causal nature of the propagation.



Probability matrices: left, at $\psi_p = 0.05$ $(r/a = 0.53)$, right $\psi_p = 0.08$ $(r/a = 0.74)$.
The vertical axis is the flight distance
The horizontal axis is the flight time.

The probability of flight distance must be normalized through

$$\int_0^\infty d\tau \int_0^1 dr p(r', r - r'; \tau) = 1.$$

Two fluid nonlocal Montroll equation

Take the domain of plasma to be $0 < r < 1$. Introduce passing and trapped particle densities n_p and n_t coupled together through pitch angle scattering and Lévy flights,

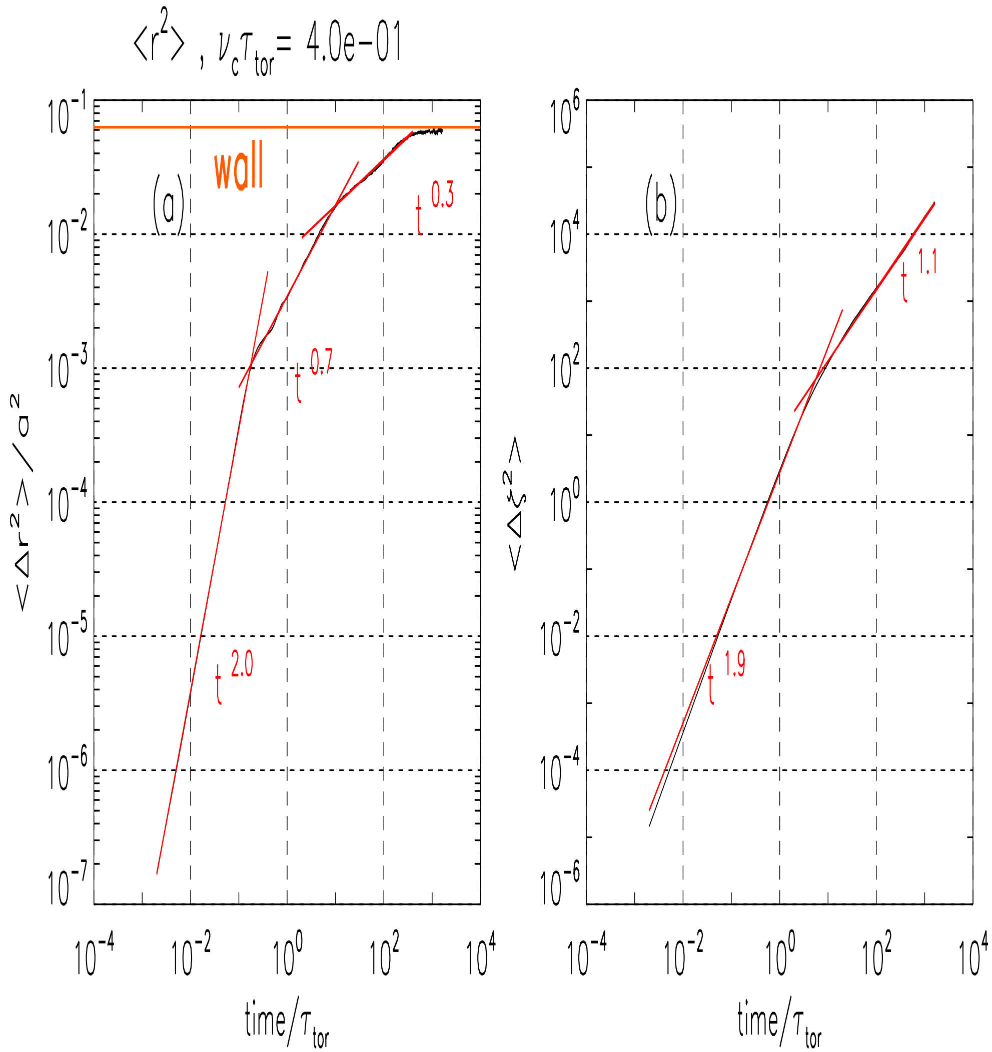
$$\begin{aligned}\partial_t n_t(r,t) =& S_t(r,t) - A(r)\nu n_t + B(r)\nu n_p + D(r)\partial_r^2 n_t \\ &+ \int_0^t dt' \int_0^1 dr' p(r', r-r'; t-t') n_p(r', t'), \\ \partial_t n_p(r,t) =& S_p(r,t) + A(r)\nu n_t - B(r)\nu n_p + D(r)\partial_r^2 n_p \\ &- \int_0^t dt' n_p(r,t')[\int_0^1 dr' p(r, r'-r; t-t')] \, .\end{aligned}$$

where $D(r)$ is the local particle diffusion rate due to neoclassical and classical scattering and S_p and S_t are local sources.

A Lévy flight of a passing particle ends when the particle becomes trapped, giving the integral in the first equation.

Integral in second equation is the loss of passing particles on flights,

Total particle number conserved in the absense of sources.



Ion transport in a typical, chaotic state in the RFX reversed-field pinch:

(a) $(r(t) - r(0))^2$ vs t

(b) $(\zeta(t) - \zeta(0))^2$ vs t

Short time streaming, subdiffusion with $p = 0.7$ and finally wall limitation. The two component Montroll equation reproduces the subdiffusive character of the transport in the RFP, giving the hollow density profiles due to nonlocal Lévy type transport.

Conclusion

- The presence of small perturbations can lead to nonlocal transport
- Both pre disruption states and TAE spectra are candidates
- A spectrum of TAE modes can give diffusive or subdiffusive transport
- $< \delta P_\zeta^2 >= Dt^p$ with $p = .5$ is common but not universal
- The Rechester-Rosenbluth formalism is often not applicable
- Spatially varying levels of chaos require local treatment
- Zaslavsky style fractional derivative analysis assumes homogeneity
- An integral Montroll type formalism can capture the local variability
- Assuming diffusion is dangerous, nonlocal transport is often present
- Directly pushing particles is always safe, they will participate in flights